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Modeling COVID-19 Pandemic using Susceptible-Infected-Recovered (SIR) Model for Karachi City

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ABSTRACT

Numerical models can help identify the peak infection time of an epidemic. In Karachi, since the detection of patient zero on 26th February the infection has spread at an exponential rate. The epidemic may reach a point when rigorous measures should be implemented. In this study Susceptible-Infected-Recovered (SIR) model is applied to predict the peak infection of COVID-19 in the population of Karachi City and compared with the number of reported cases by Sindh Population and Welfare Department's database. The model was validated with the Lahore coronavirus cases correlation coefficient of modeled and observed data for Lahore City was observed to be 0.9736. According to the model prediction, Karachi would experience peak infection on 150th day that would be 25th July 2020 since the first case was reported on 26th February 2020. The correlation coefficient of modeled and observed data for historic period of 62 days is 0.9816. Measures like social distancing and strict operating procedure for essential community services should be adopted to control this spread otherwise the number of infected may result in collapse of the medical system.

Keywords: Epidemic infection modelling, peak infection for Karachi City, SIR model.

1. Introduction

Karachi is the cosmopolitan city of Pakistan and stand in world's densely populated cities. The city is home to more than 16 million people and considered to be the economic hub of Pakistan. The first patient reported of Covid-19 was detected in Karachi was on 26th February 2020. The travel history of patient zero was traced back to Iran which was already suffering severely from the virus spread. The patient was accompanied by other possibly infected worshippers from Iran which resulted in early local transmission of virus within community. Since the detection of patient zero the virus continued to spread at exponential rate and have the second most cases reported in any city within the country. Intervention as lockdown was imposed in the city on 23rd march 2020 to limit mobility and interaction rate of citizens. However, relaxation of lockdown on 14th April 2020 the cases have continued to surge and the situation may escalate and may overburden the current health system of the city.

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This study aims to model the COVID-19 epidemic in Karachi City, the model will simulate when the peak of the infection would be observed and the when the epidemic would come to its conclusion in the Karachi City.

2. Model

Mathematical models are developed with aim to stimulate the spread of epidemic. Ever since the global outbreak of Coronavirus different mathematical models have been stimulated to predict the pattern and growth of virus and measures to take accordingly. In this study SIR model is employed to predict the virus spread in the Karachi city. The SIR model simulates the total susceptible cases (S), infected cases (I) and recovered cases (R) within a population during the virus outbreak at any instance. The SIR model is referred to as a compartmental model which models epidemic by breaking given population into Susceptible, Infected and Recovered phase. The model processes each individual of understudy population through each compartment.

2.1. Modelling Equations

The model is based on the methodology and the set of assumption reported by Ronald Ross and William Hammer [1], these are depicted in the differential equations as follow:

$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dy} = \frac{\beta IS}{N} - \gamma I \qquad \text{(Eq. 1)}$$

$$\frac{dR}{dt} = \gamma I,$$

SIR is model is based on assumption that during simulation the total individuals during every phase of susceptible, infected and recovered is equal to total population. The equation is represented as:

$$S(t) + I(t) + R(t) = N$$

The basic reproduction number ($R_\circ$) is the fraction between the individual's susceptible (β) and individual recovered (γ) and can be represented by the following equations:

$$R_\circ = \frac{\beta}{\gamma} \qquad \text{(Eq. 2)}$$

Disease-causing organism is dependent on the value of $R_\circ$ so the rate of change in an infected individual can be represented by the following equation:

$$\frac{dI}{dT} = (R_\circ \frac{S}{N} - 1)\gamma I$$

Further presumptions of the SIR model accept that if the value of $R_\circ$ comes out to be more than the fraction of total population and the number of susceptible cases at time zero then it would infer that the flare-up will turn into a full grown outbreak.

$$R_{\circ} > \frac{N}{S(0)}$$

$$\frac{dI}{dt}(0) > \frac{N}{S(0)}$$

Whether or not the virus outbreak will cause an epidemic can be deduced from the fact if the value of $R_{\circ}$ is less than $N/S_{(0)}$.

$$R_{\circ} < \frac{N}{S(0)}$$

$$\frac{dI}{dt}(0) < \frac{N}{S(0)}$$

2.2. Model Parameters

SIR model equations functions with four primary variables $R_{(0)}$, β ,γ. And $S_{(0)}$. For the case of Coronavirus, the value of $R_{\circ}$ shows high variability and differ from nation to nation. As reported by various sources [2-6], $R_{\circ}$ ranges between 1.4 – 3.9. An exact value of $R_{\circ}$ cannot be determined as the COVID-19 epidemic is still spreading's around the globe and thus the value of $R_{\circ}$ will evolve with time[2]. With an aim to predict the time of maximum infection, this study applies the minimum, maximum and average values of $R_{\circ}$ i.e. 1.4, 3.9, 1.65. The estimation of γ is dependent upon the average period of infection for Coronavirus, so γ=0.14 (14 Days). The value of β is determined from Eq. 1 and came out to be 0.196, 0.546, 0.371 respectively for minimum maximum and average values of $R_{(0)}$. Since SIR model assumes the entire population to be susceptible to infection, so $S_{(0)}$ is taken 16,000,000 which is the population of Karachi. For simulation of peak infection period the day 1 is set to the day when the primary two cases of Coronavirus were detected in Karachi. The simulation is employed to envisage the height of infection for Karachi City ranging from the day patient zero arrived.

2.3. Model Limitation

SIR model is programed to be steady state model, therefore for analysis the population is taken as static i.e. nobody being born or dying is in consideration. The SIR model is also based on assumption that infected person has disease immunity and cannot be infected again. Due to the simplified nature of the model, it do not take into account any foreign intervention like vaccination, precautionary measures and also the heterogeneity in infection. These limitations of SIR models influence the accuracy of model prediction [7].

2.4. Observed Data

Daily Covid-19 statistics reports published on the Sindh Population and Welfare Department website is explored to acquire number of cases reported for a period of 62 days, (i.e. from 26th February 2020 to 27th April 2020). The Daily situation reports from Health department Punjab was also explored to acquire the number of cases for a 57 days period, (i.e. from 15th March 2020 to 11th May 2020) and was validated with the results simulated.

3. Results

Data from Sindh Health and Population Welfare Department's for daily and cumulative cases reported in Karachi for a period of 62 days is plotted and shown in Figure1 (a) & (b). Daily reported cases in Karachi since the patient zero was observed on 26th February 2020 which corresponds to Day 0 in Fig 1. The x-axis represents the number of days since first case and the day of lockdown as an intervention implemented is represented by 27th day. The y-axis represents the number of cases. The trajectory shows exponential increase in the observed cases in Karachi since the first case arrived.

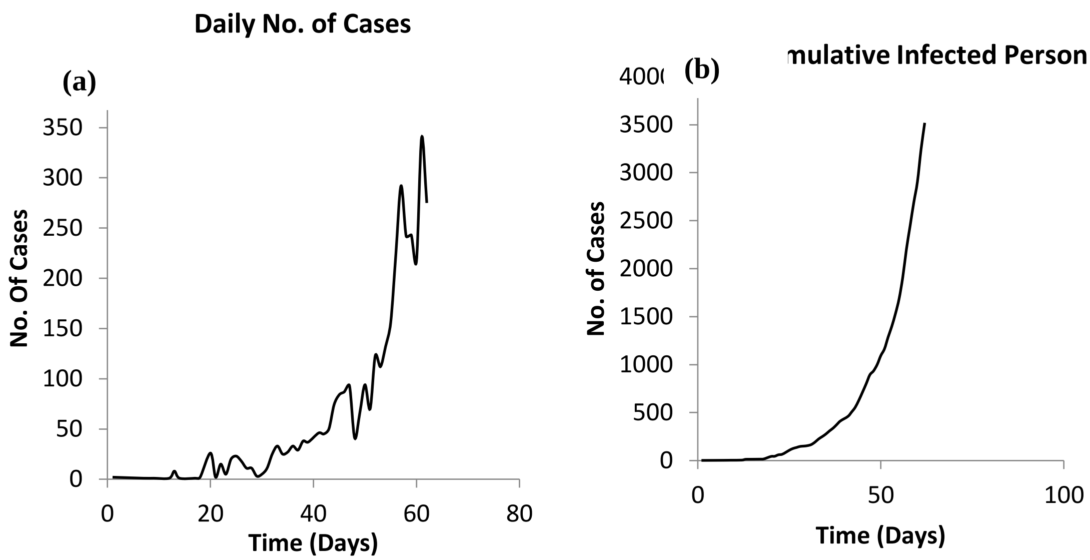

Figure 1: (a) Daily and (b) cumulative cases reported in Karachi

With the input parameters defined previously, SIR model revealed results shown in figures 2,3 and 4 as follows. Variable S, I and R represent susceptible, infected and recovered cases respectively.

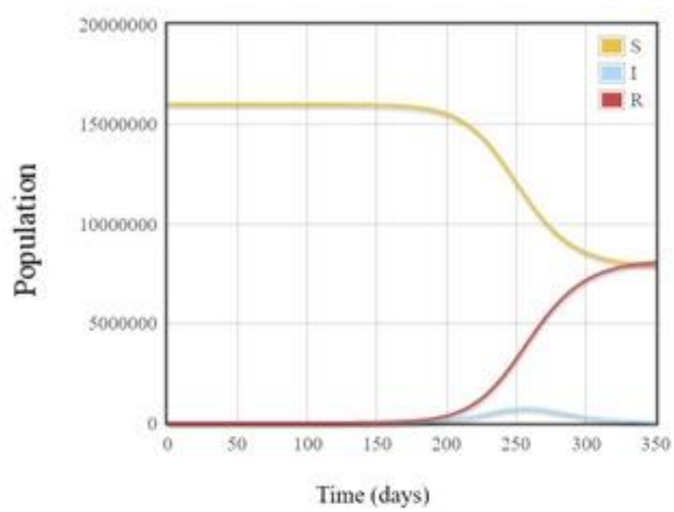

Figure 2: Model output for first set (minimum values) of input variables

With input parameters (minimum values) of β=0.0.196 and ϒ=0.14, model curve for number effected cases (blue line) is flattened which reflect decelerated infection rate. Comparing the number of observed cases with modeled ones for this set of input variables, the correlation coefficient is 0.34 only and peak infection time of COVID-19 is shown around 260th Day.

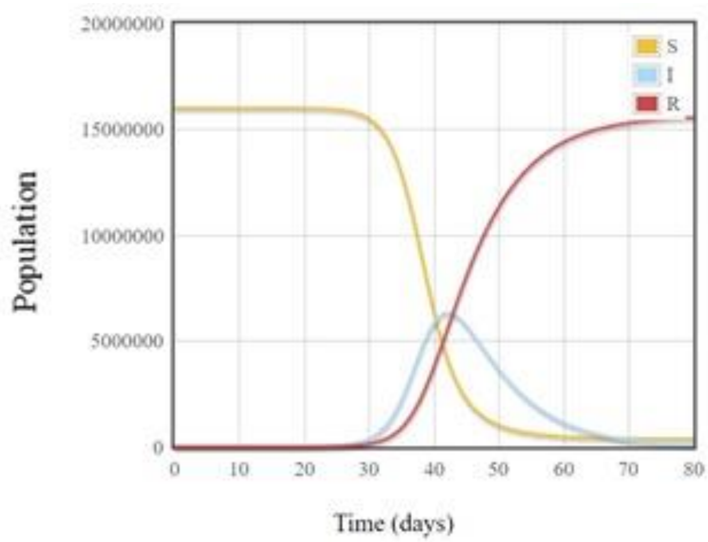

Figure 3: Model output for second set (maximum values) of input variables.

With input parameters (maximum) of β=0.546 and ϒ=0.14, results show peak to be observed on 40th day which do not correspond to the trend being experienced in Karachi. 40th Day has already elapsed by this time of model run (27th April 2020), moreover the number of populations effected is significantly high and do not comply with the observed cases.

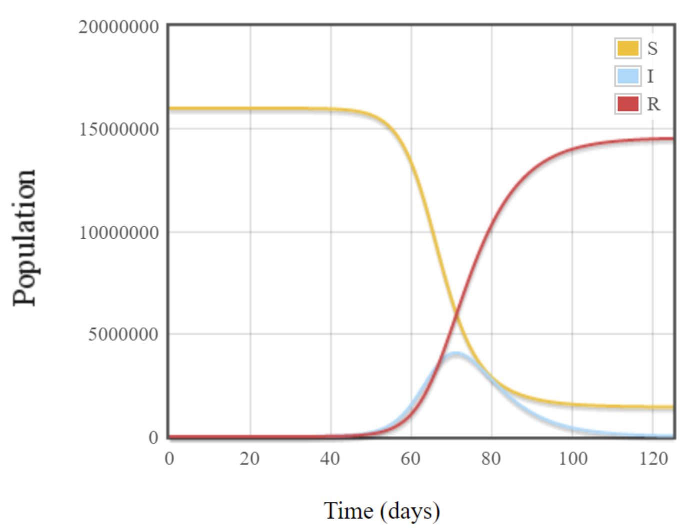

Figure 4: Model output for third set (average values) of input variables.

With input parameters (average) of β=0.371 and=ϒ 0.371, model predicted the peak infection time to be around 60th day but the number of infected persons of the simulated model and real time dataset have vast contractions. Moreover, 60th day (27th April, 2020) has already elapsed as also resulted in simulating first set of variables.

Since all the sets for input variables defined to model SIR equation do not comply with real-time situation and do not have satisfactory correlation, reverse engineering technique is utilized to simulate the model. SIR model equation is programmed to be solved with automatic solver in Microsoft Excel to result highest possible correlation with observed data. With this approach, the resulting correlation coefficient is 0.9861 and result of modeled versus observed data is shown in figure 5. This approach elucidates the resulting parameters to be accurate for the population of Karachi.

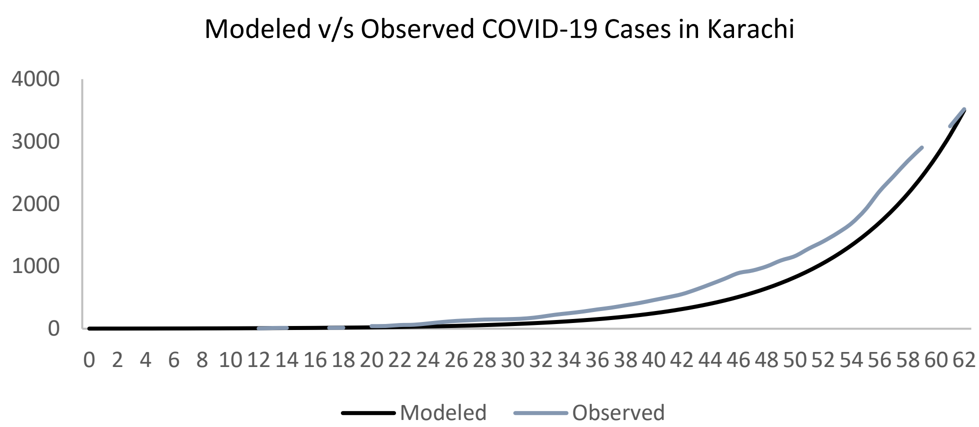

Figure 5: Modeled v/s observed COVID-19 cases using solver variables for Karachi City

The resulting parameters obtained from automatic solver in Microsoft Excel for Karachi City were further validated with simulating model for the coronavirus cases of the Lahore City. The resulting correlation is 0.9736 and the result of modeled vs observed data is shown in Figure 6. This further elucidated that the model simulation and resulting parameters to be accurate.

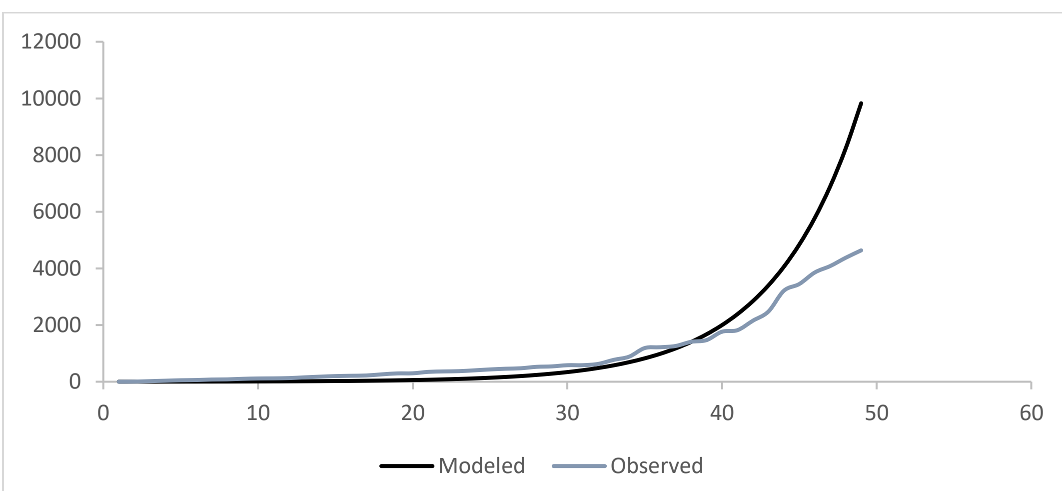

Figure 6: Modeled v/s observed COVID-19 cases using solver variables for Lahore City

Simulating SIR equations for the variables calculated from this approach revealed the infectograph shown in Figure 7.

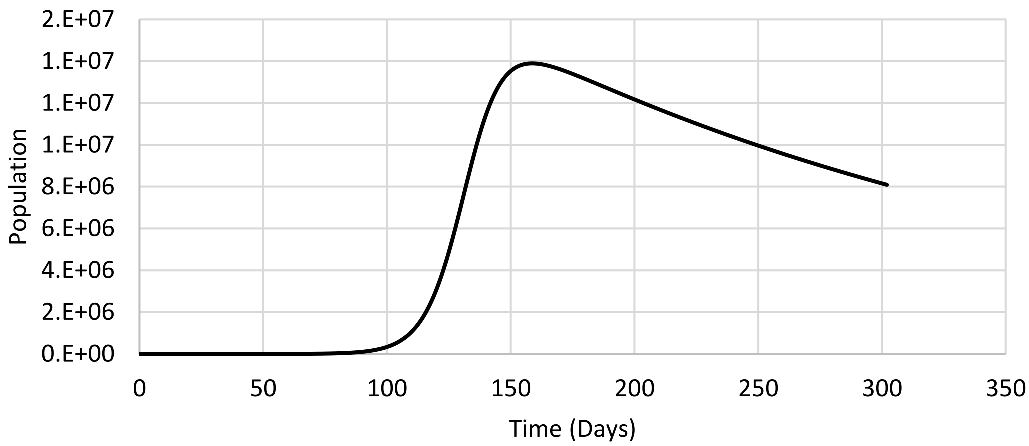

Figure 7: Model results for peak infection period

From the resulting data, it is evident that the peak infection will occur on 150th day which corresponds to 25th July 2020. The population is expected to come out of this epidemic after 300th day which corresponds to 24th December 2020. The model predictions were found to have a conclusive correlation 0.986 with real time patient data of the Karachi City.

4. Conclusion

The study aimed to model the Coronavirus outbreak in Karachi, a metropolitan city of Pakistan to estimate the peak infection day and the conclusion of the Coronavirus epidemic. The simulation parameters were adjusted as per the populace of Karachi. Model is simulated in the conditions where Coronavirus is proliferating in a close populace of 16,000,000 individuals, without the impact of any unessential factors, for example, social separating, hand washing or travel limitations. Generated data with minimum, maximum and average values of input variables, did not correlate with observed data set, reverse engineering was applied to calculate the input variables which gave significantly high correlation ($r= 0.98$, $p<0.001$). Thus, in absence of reliable data reflecting the scenario for Karachi City we assume that these outcomes can be utilized as an indicator of a presumable situation of Coronavirus for Karachi.

The results revealed that the peak infection day of the virus outbreak will occur on 150th day that would be 25th July 2020 from 26th February 2020, the day patient zero was detected. The results showed that the end-point of epidemic would be observed on 300th day that would correspond to 24th December 2020 from the day first patient was detected. It is recommended that WHO guidelines for the prevention of spread of virus should be strictly imposed. Government should impose strict lockdown as per WHO guidelines and ensures public follows protection measures against the virus. Awareness campaigns should be conducted at local laymen level to spread awareness against the virus among laymen and prompting him to follow precautionary measures.

5. Competing Interests

The authors did not find any conflicting interest and declare that there is not any conflicting interest.

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