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Proving the Truth of the Riemann Hypothesis by Introducing the Generating Function for Prime Numbers

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Abstract: The Riemann zeta function ζ(s) plays a crucial role in number theory and its applications. The Riemann Hypothesis (RH) posits that zeros of ζ(s) other than the trivial ones are located on the line defined by the equation Re(s) =1/2. This paper introduces proof of the Riemann Hypothesis. The proof employs a standard method, utilizing the eta function in place of the zeta function, under the assumption that the real part is greater than zero. The equation for the real and imaginary parts of the Riemann zeta function (eta function) is completely separated. Additionally, using a standard method and with the help of two functions ζ(s) and ζ(1-s), the real part of the root of the zeta function is obtained. To create a generator function for prime numbers in terms of b, one can solve the root of the zeta function where it equals one (i.e.,ζ(s) = 1) and obtain a relationship between b' and prime numbers.

Keywords: Riemann Zeta Function; Number Theory; Riemann's Hypothesis

1. Introduction

The Riemann Zeta Function embodies both additive and multiplicative structures in a single function, making it the most important tool in the study of prime numbers. The Riemann zeta function is crucial in number theory and has applications in physics, probability theory, and applied statistics. It is named after the German mathematician Bernhard Riemann, who discussed it in the memoir "On the Number of Primes Less Than a Given Quantity," published in 1859.[1] Riemann knew that the function equals zero for all negative even integers -2,-4,-6, etc.(referred to as trivial zeros), and that it has

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an infinite number of zeros in the critical strip of complex numbers between the lines x = 0 and x = 1. Riemann conjectured that all nontrivial zeros are on the critical line, a conjecture that later became known as the Riemann hypothesis. In 1900, the German mathematician David Hilbert referred to the Riemann Hypothesis as one of the most important questions in all of mathematics, as evidenced by its inclusion in his influential list of 23 unsolved problems that he presented to 20th-century mathematicians. [2]

2. Riemann Hypothesis

The real part of every nontrivial zero of the Riemann zeta function is 1/2. Therefore, if the hypothesis is correct, all nontrivial zeros lie on the critical line consisting of the complex numbers ($\frac{1}{2} \pm$ i b), where b is a real number and i is an imaginary unit.

2.1. Riemann Zeta Function

The Riemann zeta function can be expressed in the following form for complex s.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots + \frac{1}{n^s} + \cdots \qquad , \qquad \mathrm{Re}(s) > 1 \qquad (1)$$

The Riemann hypothesis discusses zeros outside the region of convergence of this series and Euler product. To make sense of the hypothesis, it is necessary to analytically continue the function to obtain a form that is valid for all complex s. Because the zeta function is meromorphic, all choices of how to perform this analytic continuation will lead to the same result, by the identity theorem. A first step in this continuation observes that the series for the zeta function and the Dirichlet eta function satisfy the relation within the region of convergence for both series.

$$\left(1 - \frac{2}{2^s}\right)\zeta(s) = \eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots \qquad , \qquad \mathrm{Re}(s) > 0 \qquad (2)$$

However, the zeta function series on the right converges not just when the real part of s is greater than one, but more generally whenever s has a positive real part. Thus, the zeta function can be redefined as $\eta(s)/\left(1 - \frac{2}{2^s}\right)$, extending it from Re(s) > 1 to a larger domain: Re(s) > 0, except for the points where $\left(1 - \frac{2}{2^s}\right)$ is zero.

In the strip 0 < Re(s) < 1 this extension of the zeta function satisfies the functional equation.

$$\zeta(s) = 2^s\, \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s) \qquad (3)$$

2.2. Proof of the Riemann hypothesis

2.2.1. Determining the value of “a” in ζ(s)

We start by converting relation 2 into a complex form.

$$\left(1-\frac{2}{2^{s}}\right)\zeta(s)=\eta(s)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{s}} \qquad , \qquad \mathrm{Re}(s)>0$$

s= a + ib

$$\left(1-\frac{2}{2^{s}}\right)\zeta(s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-s}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-(a+ib)}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\,n^{-ib} \tag{4}$$

By utilizing the trigonometric relationship, we can convert Riemann's zeta function from a complex form to a sinusoidal form.

$$e^{\,i\,\theta}=\cos(\theta)+i\sin(\theta)$$

$$n^{-ib}=e^{\ln(n^{-ib})}=e^{-ib.\ln(n)}=\cos[b\ln(n)]-i\sin[b\ln(n)]$$

$$\left(1-\frac{2}{2^{s}}\right)\zeta(s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\,n^{-ib}=\sum_{n=1}^{\infty}(-1)^{n+1}\{n^{-a}.\cos[b\ln(n)]-i\,n^{-a}.\sin[b\ln(n)]\}$$

$$\left(1-\frac{2}{2^{s}}\right)\zeta(s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]-i\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)] \tag{5}$$

$$\mathrm{Re}\left[\left(1-\frac{2}{2^{s}}\right)\zeta(s)\right]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)] \tag{6}$$

$$\mathrm{Im}\left[\left(1-\frac{2}{2^{s}}\right)\zeta(s)\right]=-\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)] \tag{7}$$

$$\text{If }\ \zeta(s)=0 \quad \text{then:}\quad \mathrm{Re}\left[\left(1-\tfrac{2}{2^{s}}\right)\zeta(s)\right]=0 \quad , \ \mathrm{Im}\left[\left(1-\tfrac{2}{2^{s}}\right)\zeta(s)\right]=0 \tag{8}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]=0 \tag{9}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)]=0 \tag{10}$$

First, convert equation 9 from cosine to sine, and then add equation 10 to obtain equation 11.

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin\left[\frac{\pi}{2}-b\ln(n)\right]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=0 \qquad (11)$$

By expanding equation 11 using the trigonometric relation, we obtain equation 12.

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}$$

$$\mathbf{1^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(1)\right]+\sin[b\ln(1)]\} - \mathbf{2^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(2)\right] + \sin[b\ln(2)]\} + \mathbf{3^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(3)\right]+\sin[b\ln(3)]\}-\cdots+\mathbf{n^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=0$$

$$1-\mathbf{2^{-a}}.\{2\sin\left(\frac{\pi}{4}\right)\ \cos\ [\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\{2\sin\left(\frac{\pi}{4}\right)\cos[\frac{\pi}{4}-\ b\ln(3)]\}-\ldots$$

$$+\mathbf{n^{-a}}.\{2\sin\left(\frac{\pi}{4}\right)\cos\left[\frac{\pi}{4}-\ b\ln(n)\right\}=0$$

$$1-\mathbf{2^{-a}}.\{\ \sqrt{\mathbf{2}}\ .\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\sqrt{\mathbf{2}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-\ b\ln(3)]\}-\ldots+\mathbf{n^{-a}}.\{\ \sqrt{\mathbf{2}}\cos\left[\frac{\pi}{4}-b\ln(n)\right]\}=0 \qquad (12)$$

Divide both sides of equation 12 by $\sqrt{2}$

$$(\frac{1}{\sqrt{2}})-\mathbf{2^{-a}}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\cos[\frac{\pi}{4}-\ b\ln(3)]\}-\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0 \qquad (13)$$

In the second sentence of relation 13, we make a small change because 1 minus 1 equals 0.

$$(\frac{1}{\sqrt{2}})-\mathbf{2^{-a}}.\left\{\mathbf{1}-\mathbf{1}+\cos\left[\frac{\pi}{4}-\ b\ln(2)\right]\right\}+\mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}-\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=\mathbf{0} \qquad (14)$$

$$-1=-\sqrt{2}\left\{.\mathbf{1^{-a}}\cos\left[\frac{\pi}{4}-\ b\ln(1)\right]\right\} \qquad (15)$$

With the use of 15, we will have.

$$(\frac{1}{\sqrt{2}})-\mathbf{2^{-a}}.\left\{1-\sqrt{2}\left\{\mathbf{1^{-a}}\ .\cos\left[\frac{\pi}{4}-\ b\ln(1)\right]\right\}-\cos\left[\frac{\pi}{4}-\ b\ln(2)\right]\right\}+\mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}-\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-b\ln(n)\right]\right\}=\mathbf{0}$$

Once we have defined the relationships, we can revisit the function.

$$\left\{\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}-\mathbf{2^{-a}}\right)+\ (\mathbf{2^{-a}}.\sqrt{\mathbf{2}}).\mathbf{1^{-a}}.\cos[\frac{\pi}{4}-b\ln(1)]\right\}- \mathbf{2^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(2)\right]\right\}+\mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}-\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0 \qquad (16)$$

If the expression $(\frac{1}{\sqrt{2}} - 2^{-a})$ is equal to zero and the expression $(2^{-a}.\sqrt{2})$ is equal to one, then Equation 16 becomes similar to Relation 17.

$$\mathbf{1^{-a}}.\cos[\frac{\pi}{4} - b\ln(1)]\} - \mathbf{2^{-a}}.\{\cos[\frac{\pi}{4} - b\ln(2)]\} + \mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \ldots +$$

$$\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right] = 0 \quad (17)$$

By using the following trigonometric relation

$$\cos(\alpha - \beta) = (\cos\alpha.\cos\beta) + (\sin\alpha\,.\sin\beta)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right] = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\{\cos(\frac{\pi}{4}).\cos[b\ln(n)] + \sin\left(\frac{\pi}{4}\right).\sin[b\ln(n)]\} = 0 \quad (18)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left(\frac{\pi}{4}\right).\cos[b\ln(n)] + \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin\left(\frac{\pi}{4}\right).\sin[b\ln(n)] = 0 \quad (19)$$

Since the value of "$\cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$", we will have:

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)] + \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)] = 0 \quad (20)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)] = 0 \quad (21)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin[b\ln(n)] = 0 \quad (22)$$

Therefore, $(\frac{1}{\sqrt{2}}) - 2^{-a}=0$ and $2^{-a}.\sqrt{2}=1$, then: $a=\frac{1}{2}$ (23)

To obtain a, the manipulation of function clauses was only done in the second clause, which includes the prefix 2^{-a}. The expression -1+1 is added to it, resulting in a favorable outcome that confirms the correctness of Riemann's Hypothesis. However, manipulating the remaining sentences yields new values for a, which necessitates checking if there is a root on these new lines. We start from equation 13.

$$\left(\frac{1}{\sqrt{2}}\right) - \mathbf{2^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots + \mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\}$$
$$= \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3^{-a}}.\left\{-\mathbf{1} + \mathbf{1} + \cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots$$
$$+ \mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = 0$$

$$\left\{\left(\frac{\sqrt{2}}{2}\right) - \mathbf{3}^{-\mathbf{a}}\right\} + \left\{\mathbf{3}^{-\mathbf{a}}.\sqrt{2}\right\}\left\{\mathbf{1}^{-\mathbf{a}}\cos\left[\frac{\pi}{4} - \text{ b}\ln(1)\right]\right\} - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\}$$

$$+\mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\}$$
$$= 0 \qquad (24)$$

If the expression $\left(\frac{\sqrt{2}}{2}\right) - \mathbf{3}^{-\mathbf{a}}$ equals to zero and the expression $\mathbf{3}^{-\mathbf{a}}.\sqrt{2}$ equals to one, then Relation 24 becomes similar to Relation 17.

$$\mathbf{1}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(1)\right]\right\} - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\} + \mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\}$$
$$= \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left[\frac{\pi}{4} - \text{b}\ln(n)\right] = 0 \qquad (25)$$

If $\left(\frac{\sqrt{2}}{2}\right) - 3^{-a} = 0$, $3^{-a}.\sqrt{2} = 1$ then: $3^{-a} = \frac{\sqrt{2}}{2}$, $3^{a} = \sqrt{2}$, $a.\ln(3) = \frac{1}{2}.\ln(2)$, $a = \frac{\ln(2)}{\ln(3^2)}$ (26)

Similarly, for the nth sentence we will have:

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left[\frac{\pi}{4} - \text{b}\ln(n)\right]$$
$$= \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\} + \mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\}$$
$$= 0$$

$$\left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\} + \mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots +$$

$$\mathbf{n}^{-\mathbf{a}}.\left\{-\mathbf{1} + \mathbf{1} + \cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\} = 0$$

$$\left\{\left(\frac{\sqrt{2}}{2}\right) - \mathbf{n}^{-\mathbf{a}}\right\} + \left\{\mathbf{n}^{-\mathbf{a}}.\sqrt{2}\right\}\left\{\mathbf{1}^{-\mathbf{a}}\cos\left[\frac{\pi}{4} - \text{ b}\ln(1)\right]\right\} - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\} + \mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots$$
$$+ \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\} = 0 \qquad (27)$$

If the expression $\left(\frac{\sqrt{2}}{2}\right) - \mathbf{n}^{-\mathbf{a}}$ equals to zero and the expression $\mathbf{n}^{-\mathbf{a}}.\sqrt{2}$ equals to one, then Relation 27 becomes similar to Relation 17.

$$\mathbf{1}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(1)\right]\right\} - \mathbf{2}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{b}\ln(2)\right]\right\} + \mathbf{3}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - \text{ b}\ln(n)\right]\right\}$$
$$= \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos\left[\frac{\pi}{4} - \text{b}\ln(n)\right]$$
$$= 0 \qquad (28)$$

If $\left(\frac{\sqrt{2}}{2}\right) - n^{-a} = 0$, $n^{-a}.\sqrt{2} = 1$ then: $n^{-a} = \frac{\sqrt{2}}{2}$, $n^{a} = \sqrt{2}$, $a.\ln(n) = \frac{1}{2}.\ln(2)$, $a = \frac{\ln(2)}{\ln(n^2)}$ (29)

Table 1: potential values of the real part "**s**", $0 < a \le \frac{1}{2}$, for $\zeta(s) = 0$

n	2	3	4	5	6	7	8	…	m
a	$\frac{1}{2}$	0.315464	$\frac{1}{4}$	0.215338	0.193426	0.178103	$\frac{1}{6}$	…	$\frac{\ln(2)}{\ln(m^2)}$

Graphical proof:

By plotting the function at certain points, it is easy to understand that the Zeta Riemann function has no roots at these points except for the Re(s) = 1/2.

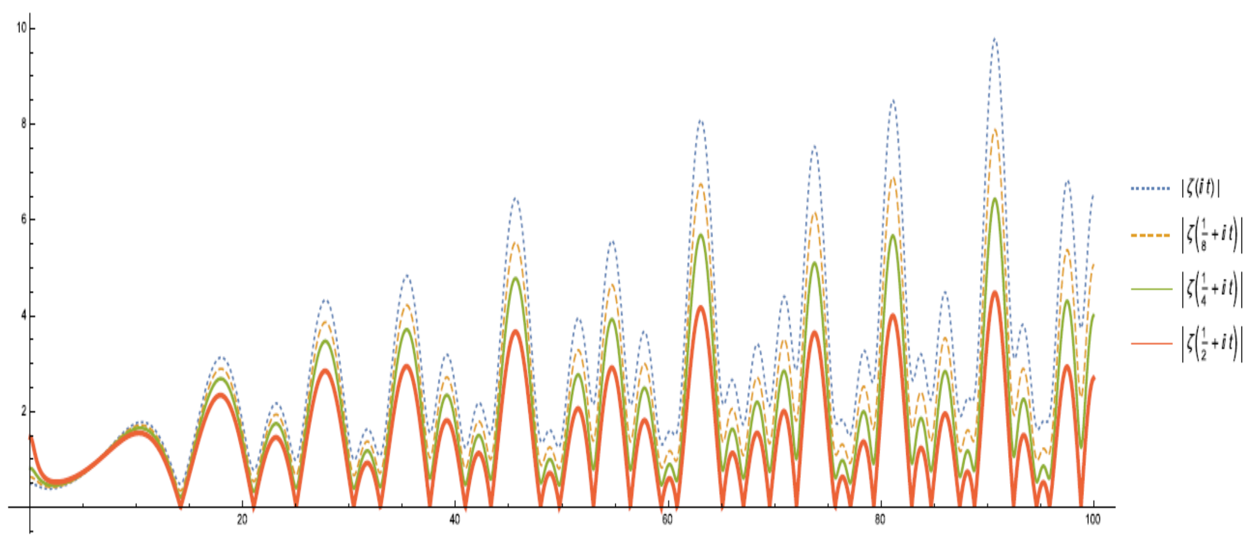

Figure 1: Plots of $|\zeta(\sigma + it)|$ with $\sigma = \left\{0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}\right\}$ and $0 \le t \le 100$

In the strip 0 < Re(s) < 1 this extension of the zeta function satisfies the functional equation.

$$\zeta(s) = \frac{1}{\left(1 - \frac{2}{2^s}\right)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{\left(1 - \frac{1}{2^{s-1}}\right)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$$

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

$\zeta(s) = \zeta(1-s)$, If $\zeta(s) = 0$, Then: $\zeta(1-s) = 0$

2.2.2. Determining the value of "a" in $\zeta(1-s)$

In this way, we can write $\zeta(1-s)$ in complex form.

$$\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)=\eta(1-s)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{1-s}} \qquad ,\mathrm{Re}(s)>0 \tag{30}$$

s= a + ib

$$\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+s}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{(-1+a+ib)}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\,n^{ib} \tag{31}$$

$e^{\,i\,\theta}=\cos(\theta)+i\sin(\theta)$

$n^{ib}=e^{\ln(n^{ib})}=e^{ib.\ln(n)}=\cos[b\ln(n)]+i\sin[b\ln(n)]$

$$\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\,n^{ib}=\sum_{n=1}^{\infty}(-1)^{n+1}\{n^{-1+a}.\cos[b\ln(n)]+i\,n^{-a}.\sin[b\ln(n)]\}$$

$$\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]+i\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+a}.\sin[b\ln(n)] \tag{32}$$

$$\mathrm{Re}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)] \tag{33}$$

$$\mathrm{Im}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+a}.\sin[b\ln(n)] \tag{34}$$

$$\text{If }\ \zeta(1-s)=0\quad\text{ then: }\quad \mathrm{Re}\left[\left(1-\tfrac{2}{2^{1-s}}\right)\zeta(1-s)\right]=0\quad,\ \mathrm{Im}\left[\left(1-\tfrac{2}{2^{1-s}}\right)\zeta(1-s)\right]=0 \tag{35}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]=0 \tag{36}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+a}.\sin[b\ln(n)]\ =0 \tag{37}$$

First, convert equation 36 from cosine to sine, and then add equation 37 to obtain equation 38.

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin\left[\frac{\pi}{2}-b\ln(n)\right]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin[b\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=0 \qquad (38)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4}-b\ln(n)\right]=0 \qquad (39)$$

By expanding equation 39 using the trigonometric relation, we obtain equation 40.

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}$$

$$\mathbf{1}^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(1)\right]+\sin[b\ln(1)]\}-\mathbf{2}^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(2)\right]+\sin[b\ln(2)]\}+\mathbf{3}^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(3)\right]+\sin[b\ln(3)]\}-\cdots+\mathbf{n}^{-1+a}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=0$$

$$1-\mathbf{2}^{-1+a}.\{2\sin\left(\frac{\pi}{4}\right)\cos\left[\frac{\pi}{4}-b\ln(2)\right]\}+\mathbf{3}^{-1+a}.\{2\sin\left(\frac{\pi}{4}\right)\cos\left[\frac{\pi}{4}-b\ln(3)\right]\}-\ldots+\mathbf{n}^{-1+a}.\{2\sin\left(\frac{\pi}{4}\right)\cos\left[\frac{\pi}{4}-b\ln(n)\right\}=0$$

$$1-\mathbf{2}^{-1+a}.\{\sqrt{\mathbf{2}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(2)]\}+\mathbf{3}^{-1+a}.\{\sqrt{\mathbf{2}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(3)]\}-\ldots+\mathbf{n}^{-1+a}.\{\sqrt{\mathbf{2}}\cos\left[\frac{\pi}{4}-b\ln(n)\right]\}=0 \qquad (40)$$

Divide both sides of equation 40 by $\sqrt{2}$.

$$\left(\frac{1}{\sqrt{2}}\right)-\mathbf{2}^{-1+a}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3}^{-1+a}.\{\cos[\frac{\pi}{4}-b\ln(3)]\}-\ldots+\mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-b\ln(n)\right]\right\}=0 \qquad (41)$$

In the second sentence of relation 41, we make a small change because 1 minus 1 equals 0.

$$\left(\frac{1}{\sqrt{2}}\right)-\mathbf{2}^{-1+a}.\left\{\mathbf{1}-\mathbf{1}+\cos\left[\frac{\pi}{4}-b\ln(2)\right]\right\}+\mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-b\ln(3)\right]\right\}-\ldots+\mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-b\ln(n)\right]\right\}=\mathbf{0} \qquad (42)$$

$$-1=-\sqrt{2}\left\{\mathbf{1}^{-\mathbf{1}+\mathbf{a}}\cos\left[\frac{\pi}{4}-b\ln(1)\right]\right\} \qquad (43)$$

With the use of 43, we will have more options.

$$\left(\frac{1}{\sqrt{2}}\right)-\mathbf{2}^{-1+a}.\left\{1-\sqrt{2}\left\{\mathbf{1}^{-\mathbf{a}}.\cos\left[\frac{\pi}{4}-b\ln(1)\right]\right\}-\cos\left[\frac{\pi}{4}-b\ln(2)\right]\right\}+\mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-b\ln(3)\right]\right\}-\ldots+\mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-b\ln(n)\right]\right\}=\mathbf{0}$$

Once we have defined the relationships, we can revisit the function.

$$\left\{\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}-\mathbf{2}^{-1+a}\right)+(\mathbf{2}^{-1+a}.\sqrt{\mathbf{2}}).\mathbf{1}^{-\mathbf{a}}.\cos[\frac{\pi}{4}-b\ln(1)]\right\}-$$

$$\mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(2)\right]\right\}+\mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}-\ldots+\ \mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0 \qquad (44)$$

If the expression $\frac{1}{\sqrt{2}}-\mathbf{2}^{-1+a}$ equals to zero and the expression $\mathbf{2}^{-1+a}.\sqrt{2}$ equals to one, then Relation 44 becomes similar to Relation 45.

$$\mathbf{1}^{-1+a}.\cos[\frac{\pi}{4}-b\ln(1)]\}-\ \mathbf{2}^{-1+a}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}-\ldots+$$

$$\mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4}-b\ln(n)\right]$$
$$=0 \qquad (45)$$

By using the following trigonometric relation

$$\cos(\alpha-\beta)=(\cos\alpha.\cos\beta)+(\sin\alpha\,.\sin\beta)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4}-b\ln(n)\right]$$
$$=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\{\cos(\frac{\pi}{4}).\cos[b\ln(n)]+\sin\left(\frac{\pi}{4}\right).\sin[b\ln(n)]\}=0 \qquad (46)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left(\frac{\pi}{4}\right).\cos[b\ln(n)]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin\left(\frac{\pi}{4}\right).\sin[b\ln(n)]=0 \qquad (47)$$

Since the value of "$\cos(\frac{\pi}{4})=\sin(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$", we will have:

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]+\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin[b\ln(n)]=0 \qquad (48)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]=0 \qquad (49)$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\sin[b\ln(n)]=0 \qquad (50)$$

Therefore, $(\frac{1}{\sqrt{2}}) - \mathbf{2}^{-1+a}=0$ and $\mathbf{2}^{-1+a}.\sqrt{2}=1$, then: $a=\frac{1}{2}$ (51)

To obtain a, the manipulation of function clauses was only done in the second clause, which includes the prefix 2^{-a}. The expression -1+1 is added to it, resulting in a favorable outcome that confirms the correctness of Riemann's Hypothesis. However, manipulating the remaining sentences yields new values for a, which necessitates checking if there is a root on these new lines. We start from Relation 41.

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right]$$
$$= \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots$$
$$+ \mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\}$$
$$= \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3}^{-1+a}.\left\{-\mathbf{1} + \mathbf{1} + \cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots$$
$$+ \mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = 0$$

$$\left\{\left(\frac{\sqrt{2}}{2}\right) - \mathbf{3}^{-1+a}\right\} + \left\{\mathbf{3}^{-1+a}.\sqrt{2}\right\}\left\{\mathbf{1}^{-1+a}\cos\left[\frac{\pi}{4} - b\ln(1)\right]\right\} - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\}$$
$$+ \mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\}$$
$$= 0 \quad (52)$$

If the expression $\left(\frac{\sqrt{2}}{2}\right) - \mathbf{3}^{-1+a}$ equals to zero and the expression $\mathbf{3}^{-1+a}.\sqrt{2}$ equals to one, then Equation 52 becomes similar to Relation 45.

If $\left(\frac{\sqrt{2}}{2}\right) - 3^{-1+a}=0$, $3^{-1+a}.\sqrt{2}=1$ then: $3^{-1+a}=\frac{\sqrt{2}}{2}$, $3^{a}=\frac{3\sqrt{2}}{2}$, $a.\ln(3)=\frac{1}{2}.\ln(2)+\ln\left(\frac{3}{2}\right)$,

$$a=\frac{\ln(2)}{\ln(3^2)} + \ln\left(\frac{3}{2}\right)/\ln(3) \quad (53)$$

Similarly, for the nth sentence we will have: #

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right]$$
$$= \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots$$
$$+ \mathbf{n}^{-\mathbf{a}}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = 0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right] = \left(\frac{\sqrt{2}}{2}\right) - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\} + \mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots +$$
$$\mathbf{n}^{-1+a}.\left\{-\mathbf{1} + \mathbf{1} + \cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = 0 \quad (54)$$

$$\left\{\left(\frac{\sqrt{2}}{2}\right) - \mathbf{n}^{-1+a}\right\} + \left\{\mathbf{n}^{-1+a}.\sqrt{2}\right\}\left\{\mathbf{1}^{-1+a}\cos\left[\frac{\pi}{4} - b\ln(1)\right]\right\} - \mathbf{2}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(2)\right]\right\}$$
$$+ \mathbf{3}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(3)\right]\right\} - \cdots + \mathbf{n}^{-1+a}.\left\{\cos\left[\frac{\pi}{4} - b\ln(n)\right]\right\} = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right]$$
$$= 0 \quad (55)$$

If the expression $\left(\frac{\sqrt{2}}{2}\right) - \mathbf{n}^{-1+a}$ equals to zero and the expression $\mathbf{n}^{-1+a}.\sqrt{2}$ equals to one, then Equation 55 becomes similar to Relation 45.

If $\left(\frac{\sqrt{2}}{2}\right) - \mathbf{n}^{-1+a} = 0$, $\mathbf{n}^{-1+a}.\sqrt{2} = 1$ then: $\mathbf{n}^{-1+a} = \frac{\sqrt{2}}{2}$, $n^a = n\frac{\sqrt{2}}{2}$,$a.\ln(n) = \frac{1}{2}.\ln(2) + \ln\left(\frac{n}{2}\right)$

$$a = \frac{\ln(2)}{\ln(n^2)} + \ln\left(\frac{n}{2}\right) / \ln(n) \quad (56)$$

Table 2: potential values of the real part "**s**", $\frac{1}{2} \leq a < 1$ for $\zeta(1-s) = 0$

n	2	3	4	…	8	…	m
a	$\frac{1}{2}$	0.684534	$\frac{3}{4}$	…	$\frac{5}{6}$	…	$a = \frac{\ln(2)}{\ln(m^2)} + \ln\left(\frac{m}{2}\right) / \ln(m)$ or $a = 1 - \frac{\ln(2)}{\ln(m^2)}$

Analytical proof:

According to Table 2, the root of ζ(1-s) lies between 1/2 and 1 if 0 < Re(s) ≤ 1/2, which is not possible. Therefore, values of a ≠ 1/2 cannot be the real part of the root of the zeta function. Thus, the function only has roots on the line Re(s) = 1/2. On the other hand, comparing of Table 1 and Table 2, shows that the only common root between ζ(s) and ζ(1-s) is a=1/2.

2.2.3. The final proof of the Riemann hypothesis

The complex form of equations (5 and 32) for $\zeta(s)$ and $\zeta(1-s)$ are written.

$$\left(1 - \frac{2}{2^s}\right)\zeta(s) = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)] - i\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-a}.\sin[b\ln(n)]$$

$$\left(1 - \frac{2}{2^{1-s}}\right)\zeta(1-s) = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)] + i\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+a}.\sin[b\ln(n)]$$

$$\zeta(s) = 0$$

$$\mathrm{Re}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = 0 \quad , \quad \mathrm{Im}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = 0$$

$$\mathrm{Re}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)] = 0$$

$$\mathrm{Im}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = \sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-a}.\sin[b\ln(n)] = 0$$

$$\zeta(1-s) = 0$$

$$\mathrm{Re}\left[\left(1 - \frac{2}{2^{1-s}}\right)\zeta(1-s)\right] = 0 \quad , \quad \mathrm{Im}\left[\left(1 - \frac{2}{2^{1-s}}\right)\zeta(1-s)\right] = 0$$

$$\mathrm{Re}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+a}.\cos[b\ln(n)]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+2a}.n^{-a}\cos[b\ln(n)]=0$$

$$\mathrm{Im}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+a}.\sin[b\ln(n)]\;=\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+2a}.n^{-a}.\sin[b\ln(n)]=0$$

$$\zeta(s)=\zeta(1-s)=0$$

$$\mathrm{Re}\left[\left(1-\frac{2}{2^{s}}\right)\zeta(s)\right]=\mathrm{Re}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]=\sum_{n=1}^{\infty}(-1)^{n+1}n^{-1+2a}.n^{-a}\cos[b\ln(n)]=0 \tag{57}$$

$$\mathrm{Im}\left[\left(1-\frac{2}{2^{s}}\right)\zeta(s)\right]=\mathrm{Im}\left[\left(1-\frac{2}{2^{1-s}}\right)\zeta(1-s)\right]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-a}.\sin[b\ln(n)]=\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-1+2a}.n^{-a}.\sin[b\ln(n)]=0 \tag{58}$$

By comparing both sides of equations 57 and 58, we can determine the value of "a".

$$n^{-1+2a}\;=1\,,\quad -1+2a=0\;\;,\quad a=\tfrac{1}{2} \tag{59}$$

Additionally, both the positive and negative values of b can be used in equations 9 and 10.

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]=0 \qquad , \qquad \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[(-b)\ln(n)]=0$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin\,[b\ln(n)]=0 \qquad , \qquad \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\sin\,[(-b)\ln(n)]=0$$

Therefore, both b and -b are valid for the function. The general solution to the equation will be $s=\frac{1}{2}\pm ib$.

2.2.4. Determining the value of "b"

If we repeat this entire process with relation 1, we will obtain the same result with slight changes in the details (Shown in ***Appendix I***). Therefore, in the next section, regardless of the discussion of convergence, to obtain the generating function of prime numbers, we will utilize relation 1 and the relation related to the zeta function and prime numbers (Euler's relation).

$$\zeta(s)=\sum_{n=1}^{\infty}n^{-s}=\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right) \qquad ,s=\frac{1}{2}+ib$$

$$\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right)=\frac{1}{1-\frac{1}{P^{S}}}=\frac{P^{S}}{P^{S}-1}=\frac{P^{\frac{1}{2}+ib}}{P^{\frac{1}{2}+ib}-1}=\frac{P^{ib}}{P^{ib}-\frac{1}{\sqrt{P}}}$$

$$P^{ib} = e^{\ln(P^{ib})} = e^{ib.\ln(P)} = \cos[b\ln(P)] + i\sin[b\ln(P)]$$

$$\frac{P^{ib}}{P^{ib} - \frac{1}{\sqrt{P}}} = \frac{\left(1 - \frac{\cos(b\ln(P))}{\sqrt{P}}\right) + i\left(-\frac{\sin(b\ln(P))}{\sqrt{P}}\right)}{\left(1 + \frac{1}{P} - \frac{2\cos(b\ln(P))}{\sqrt{P}}\right)} \quad (60)$$

To determine the root of the equation, we set the value of ζ(s) equal to zero and simplify the equation.

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod\left(\frac{1}{1 - \frac{1}{ps}}\right) = \prod\left(\frac{\left(1 - \frac{\cos(b\ln(P))}{\sqrt{P}}\right) - i\left(\frac{\sin(b\ln(P))}{\sqrt{P}}\right)}{(1 + \frac{1}{P} - \frac{2\cos(b\ln(P))}{\sqrt{P}})}\right) = 0 \quad (61)$$

When solving the equation, terms with a factor of $\frac{1}{\sqrt{P}}$ are placed on the left, while the remaining terms with a factor of one are placed on the right. Expressions that involve the multiplication of multiple primes in the denominator of the fraction are ignored with high confidence, compared to expressions that have only one prime in the denominator. The complete proof of the relationship between prime numbers and the generalized zeta function is given in ***Appendix II***.

Therefore, the final form of the equation will be as follows.

$$\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\cos[b\ln(P)] + i\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 1 \quad (62)$$

$$\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\cos[b\ln(P)] = 1 \quad , \qquad \sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 0 \quad (63)$$

2.2.5. Results

The correctness of Riemann's hypothesis has been proven by accurately determining that a=1/2. The real part of every nontrivial zero of the Riemann zeta function is Re(s) = 1/2. Thus, the hypothesis is correct, and all the nontrivial zeros lie on the critical line consisting of the complex numbers a ±ib, where a= 1/2 is a real number and b is the imaginary number.

In general, the following relationships hold for the zeta function.

$$\zeta(s) = 0 \quad \text{then:} \quad \mathrm{Re}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = 0 \quad , \mathrm{Im}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = 0\,, \quad s = a + ib$$

$$a = \frac{1}{2} \quad , \quad b = \text{Unknown}$$

$$\mathrm{Re}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = \sum_{n=1}^{\infty}(-1)^{n+1}n^{-a}.\cos[b\ln(n)]$$

$$\mathrm{Im}\left[\left(1 - \frac{2}{2^s}\right)\zeta(s)\right] = -\sum_{n=1}^{\infty}(-1)^{n+1}\,n^{-a}.\sin[b\ln(n)]$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}.\cos[b\ln(n)] - i\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}.\sin[b\ln(n)] = 0$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}.\cos[b\ln(n)] = 0 \quad , \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}.\sin[b\ln(n)] = 0$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\cos[b\ln(P)] + i\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 1$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\cos[b\ln(P)] = 1 \quad , \qquad \sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 0$$

3. The generator function of prime numbers

First, we set the value of the original zeta function to 1. Using the trigonometric relationship, we convert it into a complex form and consider the real part as 1 and the imaginary part as 0. By summing the two real and imaginary components, we reach a value of a = 1/2.To find b′, we utilize the multiplicative form of prime numbers and set the value to 1 resulting in a new sinusoidal form of the real and imaginary parts which includes two parameters b′ and P. In this case, the amplitude of the zeta function is 1. With the correct assumption, the true value can be considered equal to the cosine of the arbitrary angle theta, and its imaginary part equal to the sine of the same angle. By using the relationship between the sine and cosine of the theta angle and solving the resulting equation, we obtained a correct relationship between b′ and the prime number corresponding to it.

The Riemann zeta function can be expressed in the following form for complex s.

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \sum_{n=1}^{\infty} n^{-(a+ib)} = \sum_{n=1}^{\infty} n^{-a}.n^{-ib} \qquad (64)$$

s= a + ib

By utilizing the trigonometric relationship provided, we are able to convert the shape of Riemann’s zeta function from complex to sinusoidal form.

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$n^{-ib} = e^{\ln(n^{-ib})} = e^{-ib.\ln(n)} = \cos[b\ln(n)] - i\sin[b\ln(n)]$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-a}.n^{-ib} = \sum_{n=1}^{\infty}\{n^{-a}.\cos[b\ln(n)] - i\,n^{-a}.\sin[b\ln(n)]\}$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)] - i\sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)]$$

3.1. Determining the value of a′

$\zeta(s) = 1$ then: $\mathrm{Re}[\zeta(s)] = 1$, $\mathrm{Im}[\zeta(s)] = 0$, s' = a' + ib'

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] - i\sum_{n=1}^{\infty} n^{-a'}.\sin[b'\ln(n)] = 1 \qquad (65)$$

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] = 1 \qquad , \qquad \sum_{n=1}^{\infty} n^{-a'}.\sin[b'\ln(n)] = 0 \qquad (66a, 66b)$$

We add the two relations 66a and 66b together to obtain relation 67.

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] = \sum_{n=1}^{\infty} n^{-a'}.\sin\left[\frac{\pi}{2} - b'\ln(n)\right] = 1$$

$$\sum_{n=1}^{\infty} n^{-a'}.\sin[b'\ln(n)] = 0$$

$$\sum_{n=1}^{\infty} n^{-a'}.\sin\left[\frac{\pi}{2} - b'\ln(n)\right] + \sum_{n=1}^{\infty} n^{-a}.\sin[b'\ln(n)] = 1$$

$$\sum_{n=1}^{\infty} n^{-a'}.\left\{\sin[b'\ln(n)] + \sin\left[\frac{\pi}{2} - b'\ln(n)\right]\right\} = 1 \qquad (67)$$

By expanding equation 67 using the trigonometric relation, we obtain equation 68.

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}$$

$$\sum_{n=1}^{\infty} n^{-a'}.\left\{\sin[b'\ln(n)] + \sin\left[\frac{\pi}{2} - b'\ln(n)\right]\right\}$$

$$= 2\sum_{n=1}^{\infty} n^{-a'}.\sin\left[\frac{\pi}{4}\right].\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \sqrt{2}\sum_{n=1}^{\infty} n^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = 1$$

$$\sum_{n=1}^{\infty} n^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \frac{\sqrt{2}}{2}$$

$$\mathbf{1}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(1)\right] + \mathbf{2}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(2)\right] + \mathbf{3}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(3)\right] + \ldots + \mathbf{n}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} + 2^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(2)\right] + 3^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(3)\right] + \ldots + n^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \frac{\sqrt{2}}{2} \qquad (68)$$

In the second sentence of relation 68, we make a small change because -1+1 equals 0.

$$\frac{\sqrt{2}}{2} + \mathbf{2}^{-a'}\left\{-1+1+\cos\left[\frac{\pi}{4} - b'\ln(2)\right]\right\} + \mathbf{3}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(3)\right] + \cdots + \mathbf{n}^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \frac{\sqrt{2}}{2}$$

$$1 = \sqrt{2}\,\{\mathbf{1}^{-a'}.\cos[\tfrac{\pi}{4} - b'\ln(1)]\} \tag{69}$$

With the use of 71, we will have it.

$$\{\frac{\sqrt{2}}{2} - 2^{-a'}\} + \{(2^{-a'}.\sqrt{2}).\mathbf{1}^{-a'}.\cos[\tfrac{\pi}{4} - b'\ln(1)]\} + \mathbf{2}^{-a'}.\left\{\cos\left[\tfrac{\pi}{4} - b'\ln(2)\right]\right\} + \mathbf{3}^{-a'}\left\{\cos\left[\tfrac{\pi}{4} - b'\ln(3)\right]\right\} + \ldots$$

$$+\mathbf{n}^{-a'}.\left\{\cos\left[\frac{\pi}{4} - b'\ln(n)\right]\right\} = \frac{\sqrt{2}}{2} \tag{70}$$

If the expression $(\frac{1}{\sqrt{2}} - 2^{-a})$ is equal to zero and the expression $(2^{-a}.\sqrt{2})$ is equal to one, then Relation 70 becomes similar to Relation 68.

$$\mathbf{1}^{-a'}.\cos[\tfrac{\pi}{4} - b'\ln(1)]\} + \mathbf{2}^{-a'}.\left\{\cos\left[\tfrac{\pi}{4} - b'\ln(2)\right]\right\} + \mathbf{3}^{-a'}\left\{\cos\left[\tfrac{\pi}{4} - b'\ln(3)\right]\right\} + \ldots$$

$$+\mathbf{n}^{-a'}.\left\{\cos\left[\frac{\pi}{4} - b'\ln(n)\right]\right\} = \sum_{n=1}^{\infty} n^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right] = \frac{\sqrt{2}}{2}$$

By using the following trigonometric relation

$$\cos(\alpha - \beta) = (\cos\alpha.\cos\beta) + (\sin\alpha\,.\sin\beta)$$

$$\sum_{n=1}^{\infty} n^{-a'}.\cos\left[\frac{\pi}{4} - b'\ln(n)\right]$$

$$= \sum_{n=1}^{\infty} n^{-a'}.\{\cos(\frac{\pi}{4}).\cos[b'\ln(n)] + \sin\left(\frac{\pi}{4}\right).\sin[b'\ln(n)]\} = \frac{\sqrt{2}}{2} \tag{71}$$

$$\sum_{n=1}^{\infty} n^{-a'}.\cos\left(\frac{\pi}{4}\right).\cos[b'\ln(n)] + \sum_{n=1}^{\infty} n^{-a'}.\sin\left(\frac{\pi}{4}\right).\sin[b'\ln(n)] = \frac{\sqrt{2}}{2} \tag{72}$$

Since the value of "$\cos(\frac{\pi}{4}) =$ "$\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$", we will have:

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] + \sum_{n=1}^{\infty} n^{-a'}.\sin[b'\ln(n)] = 1 \tag{73}$$

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] = 1 \quad , \quad \sum_{n=1}^{\infty} n^{-a'}.\sin[b'\ln(n)] = 0 \tag{66a, 66b}$$

Therefore, $\frac{\sqrt{2}}{2} - 2^{-a'} = 0$ and $2^{-a'}.\sqrt{2} = 1$, then: $a' = \frac{1}{2}$ (74)

Additionally, both the positive and negative values of b can be applied in equations 66.

$$\sum_{n=1}^{\infty} n^{-a'}.\cos[b'\ln(n)] = 1 \quad , \quad \sum_{n=1}^{\infty} n^{-a'}.\cos[(-b')\ln(n)] = 1$$

$$\sum_{n=1}^{\infty} n^{-a'}.\sin\left[b'\ln(n)\right]=0 \qquad , \qquad \sum_{n=1}^{\infty} n^{-a'}.\sin\left[(-b')\ln(n)\right]=0$$

Therefore, both b′ and -b′ are valid for the function. The general solution to the equation will be $s'=\frac{1}{2}\pm ib'$. By substituting a =1/2, relations 65 and 66 are transformed into the following relations. Then by numerically solving the equation, we can determine the values of b′.

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\cos[b'\ln(n)]-i\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\sin[b'\ln(n)]=1 \tag{75}$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\cos[b'\ln(n)]=1 \qquad , \qquad -\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\sin[b'\ln(n)]=0 \tag{76a, 76b}$$

Similar to the proof presented in Section **2.2.4**, the relation of prime numbers with the generalized zeta function is given in ***Appendix III***.

$$\zeta(s)=\sum_{n=1}^{\infty}n^{-s}=\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right) \qquad , s'=\frac{1}{2}+ib' \tag{77}$$

$$\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right)=\frac{1}{1-\frac{1}{P^{S}}}=\frac{P^{S}}{P^{S}-1}=\frac{P^{\frac{1}{2}+ib'}}{P^{\frac{1}{2}+ib'}-1}=\frac{P^{ib'}}{P^{ib'}-\frac{1}{\sqrt{P}}}$$

$$\zeta(s)=\sum_{n=1}^{\infty}n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\prod\left(\frac{\left(1-\frac{\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)-i\left(\frac{\sin\left(b'\ln(P)\right)}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)}\right)=1 \tag{78}$$

When solving the equation, terms with a factor of $\frac{1}{\sqrt{P}}$ are placed on the left, while the remaining terms with a factor of one are placed on the right. Expressions that involve the multiplication of multiple primes in the denominator of the fraction are ignored with high confidence, compared to expressions that have only one prime in the denominator.

Therefore, the final form of the equation will be as follows.

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\cos\left(b'\ln(P)\right)-i\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\sin\left(b'\ln(P)\right)=0 \tag{79}$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\cos\left(b'\ln(P)\right)=0 \quad , \qquad -\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\sin\left(b'\ln(P)\right)=0 \tag{80a, 80b}$$

3.2. Definition of the generating function of prime numbers

The real and imaginary components of equation 78 can be thought of as the cosine and sine of a trigonometric angle.

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\prod\left(\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right)=1$$

$$\text{If}\quad \cos[2\pi-\theta p]=\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\quad ,\text{Then:}\quad \sin[2\pi-\theta p]=\sqrt{1-\cos^2(2\pi-\theta p)} \tag{81}$$

$$\sin[2\pi-\theta p]=\sqrt{1-\left\{\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right\}^2}=\sqrt{\frac{\frac{1}{P}\left(1-\cos^2(b'\ln(P))\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}}=\frac{\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}$$

$$2\pi-\theta p=\cos^{-1}\left(\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right)\quad ,or\quad 2\pi-\theta p=\sin^{-1}\left(\frac{\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right) \tag{82}$$

We can use the trigonometric relationship of the sum of the squares of sine and cosine to then obtain an independent relationship between b′ and the prime number.

$$\cos^2(2\pi-\theta p)+\sin^2(2\pi-\theta p)=1\quad ,\quad \frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)^2}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}+\frac{\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)^2}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}=1$$

$$\frac{\left(1-\frac{2\cos(b'\ln(P))}{\sqrt{P}}+\frac{\cos^2(b'\ln(P))}{P}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}+\frac{\left(\frac{\sin^2(b'\ln(P))}{P}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}=1$$

$$\frac{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2}=1\quad ,\quad \left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)=\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)^2$$

$$\left(1+\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)^2-\left(1+\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)=\left(1+\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right).\left(\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)=0$$

$$1+\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}=0 \quad ,1+\frac{1}{P}=\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}} \quad , \quad \cos\left(b'\ln(P)\right)=\frac{\sqrt{P}}{2}.\left(1+\frac{1}{P}\right)>1$$

$$\left(\frac{1}{P}-\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}\right)=0 \quad ,\frac{1}{P}=\frac{2\cos\left(b'\ln(P)\right)}{\sqrt{P}}$$

$$\cos\left(b'\ln(P)\right)=\left(\frac{1}{2\sqrt{P}}\right) \ , \ b'\ln(P)=\arccos\left(\frac{1}{2\sqrt{P}}\right) \quad , \quad b'=\frac{1}{\ln(P)}\arccos\left(\frac{1}{2\sqrt{P}}\right), \quad |\zeta(s)|=1 \qquad (83)$$

3.3. Results

To find the values of b′, you can numerically solve equation 76 and then calculate the corresponding prime number using equation 83.
In general, the following relationships hold for the zeta function.

$$\zeta(s)=1 \quad \text{then:} \quad \mathrm{Re}[\zeta(s)]=1 \quad , \ \mathrm{Im}[\zeta(s)]=0 \quad , s'=a'+i\,b'$$

$$a'=\frac{1}{2} \quad , \qquad b'=\frac{1}{\ln(P)}.\arccos\left(\frac{1}{2\sqrt{P}}\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\cos[b'\ln(n)]-i\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\sin[b'\ln(n)]=1$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\cos[b'\ln(n)]=1 \quad , \qquad -\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.\sin[b'\ln(n)]=0$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\cos[b'\ln(P)]-i\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\sin[b'\ln(P)]=0$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\cos[b'\ln(P)]=0 \quad , \qquad -\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}.\sin[b'\ln(P)]=0$$

4. Conclusions

In this article, we began by attempting to prove the Riemann hypothesis. We started by working with the initial form of the function and then transformed it into its complex form. To find the roots of the function's real and imaginary values, we set it equal to zero. By considering s = a ± ib, we were able to derive the phase-shifted form of the equation using trigonometric relations. Next, we combined the real and imaginary parts of the equations (relations 9 and 10), expanded the resulting equation, and compared it with the phase-shifted state. This process led to obtaining two simple equations for values of a. Solving

these equations revealed that the value is equal to 1/2. Additionally, we applied the values of b and -b in equations 9 and 10, confirming that all roots of the equation lie on the 1/2 line, resulting in s = a $\pm$ ib.
It seems that obtaining a prime number generator through the zero root of Riemann's zeta function is not possible. To create a prime number generator function in terms of b′, one can solve the root of the zeta function where it equals one (i.e.,$\zeta(s) = 1$) and establish a relationship between b′ and prime numbers. By setting the value of zeta equal to one and s' = a' + ib′, similar to zeta equal to zero, the roots are once again placed on the 1/2 line. Moving forward, we will perform operations on equation 77, which represents the complex form of the $\prod$ function. We assume that the real component is the cosine θp function and the imaginary component is the sine θp function. By using the trigonometric relationship that the square of the sine plus the square of the cosine equals one, we can derive an independent relationship between b′ and P. Therefore, if the value of b′ can be obtained from equation 66 as a numerical solution, then by using the relationship between b′ and P, referred to as the generating function of the prime number (relation 83), the prime number corresponding to b′ can be easily obtained.

Declaration
On behalf of all authors, the corresponding author states that there is no conflict of interest.

Conflict of interest statement
We have no conflicts of interest to disclose.

Data Availability Statements

The data that support the findings of this study are openly available in [repository name] at https://www.claymath.org /[sites/default/files/ezeta.pdf], reference number [1].

References

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Appendix I

The Riemann zeta function can be expressed in the following form for complex **s**:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \sum_{n=1}^{\infty} n^{-(a+ib)} = \sum_{n=1}^{\infty} n^{-a}. n^{-ib} \quad (1)$$

$$\mathbf{s} = a + ib \quad (2)$$

By using the following trigonometric relationship, we can transform the shape of Riemann's zeta function from complex to sinusoidal form. $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$$n^{-ib} = e^{\ln(n^{-ib})} = e^{-ib.Ln(n)} = \cos[b\ln(n)] - i\sin[b\ln(n)]$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-a}. n^{-ib} = \sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] - i\, n^{-a}. \sin[b\ln(n)]$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] - i\sum_{n=1}^{\infty} n^{-a}. \sin[b\ln(n)] \quad (3)$$

$$\mathrm{Re}[\zeta(s)] = \sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] \quad (4)$$

$$\mathrm{Im}[\zeta(s)] = -\sum_{n=1}^{\infty} n^{-a}. \sin[b\ln(n)] \quad (5)$$

$$\text{if } \zeta(s) = 0 \quad \text{then:} \quad \mathrm{Re}[\zeta(s)] = 0 \quad , \ \mathrm{Im}[\zeta(s)] = 0 \quad (6)$$

$$\sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] = 0 \quad (7)$$

$$\sum_{n=1}^{\infty} n^{-a}. \sin[b\ln(n)] = 0 \quad (8)$$

Before we begin proving the hypothesis, we first obtain the Phase-shifted Riemann relation using the following trigonometric relation, which we will reference at the end of the discussion.

$$\cos(\alpha \pm \beta) = (\cos\alpha . \cos\beta) \mp (\sin\alpha . \sin\beta)$$

$$\sin(\alpha \pm \beta) = (\sin\alpha . \cos\beta) \pm (\cos\alpha . \sin\beta)$$

Multiply both equations 7 and 8 by $\cos[\theta]$ and $\sin[\theta]$, then add them together:

$$\sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] = \sum_{n=1}^{\infty} n^{-a}. \cos[b\ln(n)] . \cos[\theta] = 0 \quad (9)$$

$$\sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = \sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)].\sin[\theta] = 0 \quad (10)$$

$$\sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = \sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)].\cos[\theta] = 0 \quad (11)$$

$$\sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)] = \sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)].\sin[\theta] = 0 \quad (12)$$

$$\sum_{n=1}^{\infty} n^{-a}.\{\cos[b\ln(n)].\cos(\theta)] \mp \sin\big[[b\ln(n)].\sin(\theta)\big]\} = 0 \quad (13)$$

$$\sum_{n=1}^{\infty} n^{-a}.\cos[(\theta) \pm b\ln(n)] = 0 \quad (14)$$

For example: $\sum_{n=1}^{\infty} n^{-a}.\cos\left[\frac{\pi}{4} - b\ln(n)\right] = 0$

$$\zeta(s) = 0 \quad \text{then:} \quad Re[\zeta(s)] = 0 \quad , \quad Im[\zeta(s)] = 0 \quad (15)$$

$$\sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)] = 0 \qquad , \qquad \sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = 0$$

We combine relations 7 and 8 to obtain relation 16

$$\sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)] = \sum_{n=1}^{\infty} n^{-a}.\sin\left[\frac{\pi}{2} - b\ln(n)\right] = 0$$

$$\sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = 0$$

$$\sum_{n=1}^{\infty} n^{-a}.\cos[b\ln(n)] + \sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = 0$$

$$\sum_{n=1}^{\infty} n^{-a}.\sin\left[\frac{\pi}{2} - b\ln(n)\right] + \sum_{n=1}^{\infty} n^{-a}.\sin[b\ln(n)] = 0$$

$$\sum_{n=1}^{\infty} n^{-a}.\{\sin\left[\frac{\pi}{2} - b\ln(n)\right] + \sin[b\ln(n)]\} = 0 \quad (16)$$

By expanding the relation 16 using the following trigonometric relation, we get:

$$(\sin\alpha \pm \sin\beta) = 2\,[\sin(\alpha \pm \beta)/2].\ [\cos(\alpha \mp \beta)/2]$$

$$\mathbf{1^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(1)\right]+\sin[b\ln(1)]\}$$
$$+\mathbf{2^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(2)\right]$$
$$+\sin[b\ln(2)]\}$$
$$+\mathbf{3^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(3)\right]+\sin[b\ln(3)]\}+\cdots+\mathbf{n^{-a}}.\{\sin\left[\frac{\pi}{2}-b\ln(n)\right]+\sin[b\ln(n)]\}=0$$

$$1+\mathbf{2^{-a}}.\{2\sin\left(\frac{\pi}{4}\right).\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\{2\sin\left(\frac{\pi}{4}\right).\cos[\frac{\pi}{4}-\ b\ln(3)]\}+\ldots$$

$$+\mathbf{n^{-a}}.\{2\sin\left(\frac{\pi}{4}\right).\cos\left[\frac{\pi}{4}-\ b\ln(n)\right\}=0$$

$$1+\mathbf{2^{-a}}.\{\ \sqrt{\mathbf{2}}\ .\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\sqrt{\mathbf{2}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-\ b\ln(3)]\}+\ldots+\mathbf{n^{-a}}.\{\ \sqrt{\mathbf{2}}\cos\left[\frac{\pi}{4}-b\ln(n)\right]\}=0 \qquad (17)$$

Divide both sides of equation 17 by $\sqrt{2}$

$$(\frac{1}{\sqrt{2}})+\mathbf{2^{-a}}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\cos[\frac{\pi}{4}-\ b\ln(3)]\}+\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0 \qquad (18)$$

In the second sentence of relation 18, we make a small change because -1+1 equals 0.

$$(\frac{1}{\sqrt{2}})+\mathbf{2^{-a}}.\{\mathbf{-1}+\mathbf{1}+\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\mathbf{cos}[\frac{\boldsymbol{\pi}}{\mathbf{4}}-\ b\ln(3)]\}+\ldots+\mathbf{n^{-a}}.\left\{\mathbf{cos}\left[\frac{\boldsymbol{\pi}}{\mathbf{4}}-\ b\ln(n)\right]\right\}=\mathbf{0} \qquad (19)$$

$$1=\sqrt{2}\{\mathbf{1^{-a}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(1)]\} \qquad (20)$$

With the use of 20, we will have.

$$\{\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}-\mathbf{2^{-a}}\}+\{(\mathbf{2^{-a}}\ *\sqrt{\mathbf{2}}).\mathbf{1^{-a}}.\cos[\frac{\boldsymbol{\pi}}{\mathbf{4}}-b\ln(1)]\}+\mathbf{2^{-a}}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\{\cos[\frac{\pi}{4}-\ b\ln(3)]\}+\ldots$$

$$+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0$$

Once we have defined the relationships, we can revisit the Phase-Shifted Riemann Zeta Function, which is similar to relation #14.

$$(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}-\mathbf{2^{-a}})+(\mathbf{2^{-a}}*\sqrt{\mathbf{2}}).\mathbf{1^{-a}}.\cos[\frac{\pi}{4}-b\ln(1)]\}+$$
$$\mathbf{2^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(2)\right]\right\}+\mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}+\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0$$

If the expression $(\frac{1}{\sqrt{2}}$ - $2^{-a})$ is equal to zero and the expression $(2^{-a}*\sqrt{2})$ is equal to one, then equation becomes the Phase-Shifted Riemann Zeta Function (Relation 14).

$$\mathbf{1^{-a}}.\cos[\frac{\pi}{4}-b\ln(1)]\}+\mathbf{2^{-a}}.\{\cos[\frac{\pi}{4}-b\ln(2)]\}+\mathbf{3^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(3)\right]\right\}+\ldots+\mathbf{n^{-a}}.\left\{\cos\left[\frac{\pi}{4}-\ b\ln(n)\right]\right\}=0$$

$$\sum_{n=1}^{\infty}n^{-a}.\cos\left[\frac{\pi}{4}-b\ln(n)\right]=0 \qquad (21)$$

Therefore, $(\frac{1}{\sqrt{2}})-2^{-a}=0$ and $2^{-a}*\sqrt{2}=1$, then: $a=\frac{1}{2}$ (22)

Appendix II

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right) \qquad ,s=\frac{1}{2}+ib \tag{1}$$

$$\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right)=\frac{1}{1-\frac{1}{P^{S}}}=\frac{P^{S}}{P^{S}-1}=\frac{P^{\frac{1}{2}+ib}}{P^{\frac{1}{2}+ib}-1}=\frac{P^{ib}}{P^{ib}-\frac{1}{\sqrt{P}}}$$

$$P^{ib}=e^{\ln(P^{ib})}=e^{ib.\ln(P)}=\cos[b\ln(P)]+i\sin[b\ln(P)]$$

$$\frac{P^{ib}}{P^{ib}-\frac{1}{\sqrt{P}}}=\frac{\cos[b\ln(P)]+i\sin[b\ln(P)]}{\cos[b\ln(P)]+i\sin[b\ln(P)]-\frac{1}{\sqrt{P}}}=\frac{\cos[b\ln(P)]+i\sin[b\ln(P)]}{(\cos[b\ln(P)]-\frac{1}{\sqrt{P}})+i\sin[b\ln(P)]}$$

$$=\frac{\cos[b\ln(P)]+i\sin[b\ln(P)]}{(\cos[b\ln(P)]-\frac{1}{\sqrt{P}})+i\sin[b\ln(P)]}.\frac{(\cos[b\ln(P)]-\frac{1}{\sqrt{P}})-i\sin[b\ln(P)]}{(\cos[b\ln(P)]-\frac{1}{\sqrt{P}})-i\sin[b\ln(P)]}$$

$$=\frac{[\cos^2(b\ln(P))-\frac{\cos(b\ln(P))}{\sqrt{P}}+\sin^2(b\ln(P))]+i\,[\sin(b\ln(P)).\cos(b\ln(P))-\sin(b\ln(P)).\cos(b\ln(P))-\frac{\sin(b\ln(P))}{\sqrt{P}}]}{(\cos[b\ln(P)]-\frac{1}{\sqrt{P}})^2+(\sin[b\ln(P)])^2}$$

$$\frac{P^{ib}}{P^{ib}-\frac{1}{\sqrt{P}}}=\frac{\left(1-\frac{\cos(b\ln(P))}{\sqrt{P}}\right)+i\left(-\frac{\sin(b\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b\ln(P))}{\sqrt{P}}\right)} \tag{2}$$

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\prod\left(\frac{\left(1-\frac{\cos(b\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b\ln(P))}{\sqrt{P}}\right)}\right) \tag{3}$$

To determine the root of the equation, we set the value of ζ(s) equal to zero and simplify the equation.

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\prod\left(\frac{\left(1-\frac{\cos(b\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b\ln(P))}{\sqrt{P}}\right)}{(1+\frac{1}{P}-\frac{2\cos(b\ln(P))}{\sqrt{P}})}\right)=0 \tag{4}$$

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)$$
$$=\left(\frac{\left(1-\frac{\cos(b\ln(P1))}{\sqrt{P1}}\right)-i\left(\frac{\sin(b\ln(P1))}{\sqrt{P1}}\right)}{\left(1+\frac{1}{P1}-\frac{2\cos(b\ln(P1))}{\sqrt{P1}}\right)}\right)\left(\frac{\left(1-\frac{\cos(b\ln(P2))}{\sqrt{P2}}\right)-i\left(\frac{\sin(b\ln(P2))}{\sqrt{P2}}\right)}{\left(1+\frac{1}{P2}-\frac{2\cos(b\ln(P2))}{\sqrt{P2}}\right)}\right)$$

$$\dots\left(\frac{\left(1-\frac{\cos\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)-i\left(\frac{\sin\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)}{\left(1+\frac{1}{Pn}-\frac{2\cos\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)}\right)=0$$

$$\left(\left(1-\frac{\cos\left(b\ln(P1)\right)}{\sqrt{P1}}\right)-i\left(\frac{\sin\left(b\ln(P1)\right)}{\sqrt{P1}}\right)\right)\left(\left(1-\frac{\cos\left(b\ln(P2)\right)}{\sqrt{P2}}\right)-i\left(\frac{\sin\left(b\ln(P2)\right)}{\sqrt{P2}}\right)\right)\dots\left(\left(1-\frac{\cos\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)-i\left(\frac{\sin\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)\right)=0$$

To simplify the multiplication process, the expression is changed from complex to exponential form.

$$\left(1-\frac{\cos\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)-i\left(\frac{\sin\left(b\ln(Pn)\right)}{\sqrt{Pn}}\right)=1-\frac{1}{\sqrt{Pn}}\left[\cos\left(b\ln(Pn)\right)+i\sin\left(b\ln(Pn)\right)\right]=1-\frac{1}{\sqrt{Pn}}.e^{ib\ln(Pn)}$$

$$\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\left(1-\frac{1}{\sqrt{P1}}.e^{ib\ln(P1)}\right)\left(1-\frac{1}{\sqrt{P2}}.e^{ib\ln(P2)}\right)\left(1-\frac{1}{\sqrt{P3}}.e^{ib\ln(P3)}\right)\dots\left(1-\frac{1}{\sqrt{Pn}}.e^{ib\ln(Pn)}\right)=0$$

$$\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\left(1-\frac{1}{\sqrt{P1}}.e^{ib\ln(P1)}-\frac{1}{\sqrt{P2}}.e^{ib\ln(P2)}+\frac{1}{\sqrt{P1}.\sqrt{P2}}.e^{ib[\ln(P1)+\ln(P2)]}\right)\left(1-\frac{1}{\sqrt{P3}}.e^{ib\ln(P3)}\right)\dots\left(1-\frac{1}{\sqrt{Pn}}.e^{ib\ln(Pn)}\right)=0$$

$$\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\left(1-\frac{1}{\sqrt{P1}}.e^{ib\ln(P1)}-\frac{1}{\sqrt{P2}}.e^{ib\ln(P2)}-\frac{1}{\sqrt{P3}}.e^{ib\ln(P3)}-\dots-\frac{1}{\sqrt{Pn}}.e^{ib\ln(Pn)}\pm\frac{1}{\sqrt{P1.P3\dots Pn}}.e^{ib[\ln(P1)+\ln(P3)+\dots+\ln(Pn)]}\mp\frac{1}{\sqrt{P2.P3\dots Pn}}.e^{ib[\ln(P2)+\ln(P3)+\dots+\ln(Pn)]}\pm\dots\mp\frac{1}{\sqrt{Pn-1.Pn}}.e^{ib[\ln(Pn-1)+\ln(Pn)]}\pm\frac{1}{\sqrt{P1.P2.P3\dots Pn}}.e^{ib[\ln(P1)+\ln(P2)+\ln(P3)+\dots+\ln(Pn)]}\right)=0$$

When rearranging the terms of the equation, the terms with a factor of $\frac{1}{\sqrt{P}}$ are placed on the left side, while the remaining terms with the number one are included on the right side.

$$\frac{1}{\sqrt{P1}}.e^{ib\ln(P1)}+\frac{1}{\sqrt{P2}}.e^{ib\ln(P2)}+\frac{1}{\sqrt{P3}}.e^{ib\ln(P3)}+\dots+\frac{1}{\sqrt{Pn}}.e^{ib\ln(Pn)}=1\pm\frac{1}{\sqrt{P1.P3\dots Pn}}.e^{ib[\ln(P1)+\ln(P3)+\dots+\ln(Pn)]}\pm\frac{1}{\sqrt{P2.P3\dots Pn}}.e^{ib[\ln(P2)+\ln(P3)+\dots+\ln(Pn)]}\pm\dots\mp\frac{1}{\sqrt{Pn-1.Pn}}.e^{ib[\ln(Pn-1)+\ln(Pn)]}\pm\frac{1}{\sqrt{P1.P2.P3\dots Pn}}.e^{ib[\ln(P1)+\ln(P2)+\ln(P3)+\dots+\ln(Pn)]}$$

With high confidence, expressions involving the multiplication of several prime numbers in the denominator of the fraction can be ignored compared to expressions containing only one prime number in the denominator.

$$\frac{1}{\sqrt{\mathrm{P1}}}.\, e^{ib\ln(\mathrm{P1})} + \frac{1}{\sqrt{\mathrm{P2}}}.\, e^{ib\ln(\mathrm{P2})} + \frac{1}{\sqrt{\mathrm{P3}}}.\, e^{ib\ln(\mathrm{P3})} + \cdots + \frac{1}{\sqrt{\mathrm{Pn}}}.\, e^{ib\ln(\mathrm{Pn})} = 1 \qquad (5)$$

$$\frac{1}{\sqrt{\mathrm{P1}}}\left(\cos\left(b\ln(\mathrm{P1})\right) + i\sin\left(b\ln(\mathrm{P1})\right)\right) + \frac{1}{\sqrt{\mathrm{P2}}}\left(\cos\left(b\ln(\mathrm{P2})\right) + i\sin(b\ln(\mathrm{P2}))\right) + \cdots + \frac{1}{\sqrt{\mathrm{Pn}}}\left(\cos\left(b\ln(\mathrm{Pn})\right) + i\sin(b\ln(\mathrm{Pn}))\right) = 1$$

$$\left[\frac{1}{\sqrt{\mathrm{P1}}}\cos\left(b\ln(\mathrm{P1})\right) + \frac{1}{\sqrt{\mathrm{P2}}}\cos\left(b\ln(\mathrm{P2})\right) + \cdots + \frac{1}{\sqrt{\mathrm{Pn}}}\cos\left(b\ln(\mathrm{Pn})\right)\right] + i\left[\frac{1}{\sqrt{\mathrm{P1}}}\sin\left(b\ln(\mathrm{P1})\right) + \frac{1}{\sqrt{\mathrm{P2}}}\sin\left(b\ln(\mathrm{P2})\right) + \cdots + \frac{1}{\sqrt{\mathrm{Pn}}}\sin\left(b\ln(\mathrm{Pn})\right)\right] = 1$$

Therefore, the final form of the equation will be as follows.

$$\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\cos[b\ln(P)] + i\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 1 \qquad (6)$$

$$\sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\cos[b\ln(P)] = 1 \quad , \qquad \sum_{P=2}^{\infty} \frac{1}{\sqrt{P}}.\sin[b\ln(P)] = 0 \qquad (7)$$

Appendix III

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right) \qquad , s'=\frac{1}{2}+ib' \tag{1}$$

$$\prod\left(\frac{1}{1-\frac{1}{P^{S}}}\right)=\frac{1}{1-\frac{1}{P^{S}}}=\frac{P^{S}}{P^{S}-1}=\frac{P^{\frac{1}{2}+ib'}}{P^{\frac{1}{2}+ib'}-1}=\frac{P^{ib'}}{P^{ib'}-\frac{1}{\sqrt{P}}}$$

$$P^{ib'}=e^{\ln(P^{ib'})}=e^{ib'.\ln(P)}=\cos[b'\ln(P)]+i\sin[b'\ln(P)]$$

$$\frac{P^{ib'}}{P^{ib'}-\frac{1}{\sqrt{P}}}=\frac{\cos[b'\ln(P)]+i\sin[b'\ln(P)]}{\cos[b'\ln(P)]+i\sin[b'\ln(P)]-\frac{1}{\sqrt{P}}}=\frac{\cos[b'\ln(P)]+i\sin[b'\ln(P)]}{(\cos[b'\ln(P)]-\frac{1}{\sqrt{P}})+i\sin[b'\ln(P)]}$$

$$=\frac{\cos[b'\ln(P)]+i\sin[b'\ln(P)]}{(\cos[b'\ln(P)]-\frac{1}{\sqrt{P}})+i\sin[b'\ln(P)]}\cdot\frac{(\cos[b'\ln(P)]-\frac{1}{\sqrt{P}})-i\sin[b'\ln(P)]}{(\cos[b'\ln(P)]-\frac{1}{\sqrt{P}})-i\sin[b'\ln(P)]}$$

$$=\frac{\left(\cos^{2}(b'\ln(P))-\frac{\cos(b'\ln(P))}{\sqrt{P}}+\sin^{2}(b'\ln(P))\right)]+i\,[\sin(b'\ln(P)).\cos(b'\ln(P))-\sin(b'\ln(P)).\cos(b'\ln(P))-\frac{\sin(b'\ln(P))}{\sqrt{P}}\Big)}{(\cos[b'\ln(P)]-\frac{1}{\sqrt{P}})^{2}+(\sin[b'\ln(P)]\,)^{2}}$$

$$\frac{P^{ib}}{P^{ib}-\frac{1}{\sqrt{P}}}=\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(-\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}$$

$$\zeta(s)=\sum_{n=1}^{\infty} n^{-s}=\prod\left(\frac{1}{1-\frac{1}{ps}}\right)=\prod\left(\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right) \tag{2}$$

$$\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}$$

$$=\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)}$$

$$=\frac{1}{\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)}$$

$$\prod\left(\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(1+\frac{1}{P}-\frac{2\cos(b'\ln(P))}{\sqrt{P}}\right)}\right)$$

$$=\prod\left(\frac{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}{\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right).\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)-i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)}\right)$$

$$=\prod\left(\frac{1}{\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)}\right)=1$$

It can be concluded that

$$\prod\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)=1 \tag{3}$$

To simplify the multiplication process, the expression is converted from complex to exponential form.

$$\left(1-\frac{\cos(b'\ln(Pn))}{\sqrt{Pn}}\right)+i\left(\frac{\sin(b'\ln(Pn))}{\sqrt{Pn}}\right)=1-\frac{1}{\sqrt{Pn}}[\cos(b'\ln(Pn))-i\sin(b'\ln(Pn))]=1-\frac{1}{\sqrt{Pn}}.e^{-ib'\ln(Pn)}$$

$$\prod\left(\left(1-\frac{\cos(b'\ln(P))}{\sqrt{P}}\right)+i\left(\frac{\sin(b'\ln(P))}{\sqrt{P}}\right)\right)$$

$$=\left(1-\frac{1}{\sqrt{P1}}e^{-ib'\ln(P1)}\right)\left(1-\frac{1}{\sqrt{P2}}e^{-ib'\ln(P2)}\right)\left(1-\frac{1}{\sqrt{P3}}e^{-ib'\ln(P3)}\right)\ldots\left(1-\frac{1}{\sqrt{Pn}}e^{-ib'\ln(Pn)}\right)=1$$

$$\left(1-\frac{1}{\sqrt{P1}}.e^{-ib'\ln(P1)}-\frac{1}{\sqrt{P2}}.e^{-ib'\ln(P2)}+\frac{1}{\sqrt{P1.P2}}.e^{-ib'[\ln(P1)+\ln(P2)]}\right)\left(1-\frac{1}{\sqrt{P3}}.e^{-ib'\ln(P3)}\right)\ldots\left(1-\frac{1}{\sqrt{Pn}}.e^{-ib'\ln(Pn)}\right)$$

$$=\left(1-\frac{1}{\sqrt{P1}}e^{-ib\ln(P1)}-\frac{1}{\sqrt{P2}}e^{-ib'\ln(P2)}-\frac{1}{\sqrt{P3}}e^{-ib'\ln(P3)}-\cdots-\frac{1}{\sqrt{Pn}}.e^{-ib'\ln(Pn)}\right.$$
$$\pm\frac{1}{\sqrt{P1.P3\ldots Pn}}e^{-ib'[\ln(P1)+\ln(P3)+\cdots+\ln(Pn)]}\mp\frac{1}{\sqrt{P2.P3\ldots Pn}}e^{-ib'[\ln(P2)+\ln(P3)+\cdots+\ln(Pn)]}\pm\cdots$$
$$\left.\mp\frac{1}{\sqrt{Pn-1.Pn}}e^{-ib'[\ln(Pn-1)+\ln(Pn)]}\pm\frac{1}{\sqrt{P1.P2.P3\ldots Pn}}e^{-ib'[\ln(P1)+\ln(P2)+\ln(P3)+\cdots+\ln(Pn)]}\right)=1$$

When rearranging the terms of the equation, the terms with a factor of $\frac{1}{\sqrt{P}}$ are placed on the left side, while the remaining terms with a factor one are included on the right side.

With high confidence, expressions involving the multiplication of several prime numbers in the denominator of the fraction can be ignored when compared to expressions containing only one prime number in the denominator.

$$\frac{1}{\sqrt{P1}}.e^{-ib'\ln(P1)}+\frac{1}{\sqrt{P2}}.e^{ib'\ln(P2)}+\frac{1}{\sqrt{P3}}.e^{-ib'\ln(P3)}+\cdots+\frac{1}{\sqrt{Pn}}.e^{-ib'\ln(Pn)}=0 \tag{4}$$

$$\frac{1}{\sqrt{P1}}\big(\cos\big(b'\ln(P1)\big) - i\sin\big(b'\ln(P1)\big)\big) + \frac{1}{\sqrt{P2}}\big(\cos\big(b'\ln(P2)\big) - i\sin\big(b'\ln(P2)\big)\big)$$
$$+ \frac{1}{\sqrt{P3}}\big(\cos\big(b'\ln(P3)\big) - i\sin\big(b'\ln(P3)\big)\big) + \cdots + \frac{1}{\sqrt{Pn}}\big(\cos\big(b'\ln(Pn)\big) - i\sin\big(b'\ln(Pn)\big)\big)$$
$$= \left(\frac{1}{\sqrt{P1}}\cos\big(b'\ln(P1)\big) + \frac{1}{\sqrt{P2}}\cos\big(b'\ln(P2)\big) + \cdots + \frac{1}{\sqrt{Pn}}\cos\big(b'\ln(Pn)\big)\right)$$
$$- i\left(\frac{1}{\sqrt{P1}}\sin\big(b'\ln(P1)\big) + \frac{1}{\sqrt{P2}}\sin\big(b'\ln(P2)\big) + \cdots + \frac{1}{\sqrt{Pn}}\sin\big(b'\ln(Pn)\big)\right) = 0$$

Therefore, the final form of the equation will be as follows.

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\cos\big(b'\ln(P)\big) - i\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\sin\big(b'\ln(P)\big) = 0 \qquad (5)$$

$$\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\cos\big(b'\ln(P)\big) = 0 \quad , \qquad -\sum_{P=2}^{\infty}\frac{1}{\sqrt{P}}\sin\big(b'\ln(P)\big) = 0$$